Considering theoretical diversity and networking activities in mathematics education from a sociological point of view

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The paper focuses on the social dimensions of the issues addressed in this working group, social being considered at different levels: interactions, culture, and institutions. It addresses the following questions: what is a theoretical framework? Why are theories so numerous in mathematics education? Is it necessary to reduce this multiplicity? Why or why not? The reflection is based on the anthropological theory of the didactic (ATD) and on Bourdieu's theory of social fields. Assuming that the latter is not necessarily well-known in the mathematics education community, and that it offers an interesting potential to enrich the debate within the networking semiosphere, I devote a substantial part of my text to give an idea about the way Bourdieu applies his theory to science.

Keywords: Praxeology, paradigm, institutional determination, symbolic capital.

INTRODUCTION

Addressing the topic of theoretical diversity in mathematics education from a social point of view is not something new in the European research community. The central preoccupation in this WG has been, since CERME4, the barrier to effective communication created by the multiplicity of theories, be it communication within the field or with external partners from policy makers to educative professionals. Radford (2008, p. 318) suggests considering the networking practices as located in a *semiosphere*, that is, "an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other". It is quite representative of the interaction dimension in networking activities. Among the social aspects this paper considers, some have been more erratically present in the discussions. For instance, the WG11 leaders' introduction at CERME4 (Artigue et al., 2006, p. 1240) refers to a theoretical "more intrinsic diversity linked to the diversity of educational cultures and to the diversity of the institutional characteristics of the development of the field in mathematics education in different countries or global areas." This issue of theoretical multiplicity linked to cultural diversity has not recently been discussed in CERME. Yet the influence of cultural contexts on the research in mathematics education has been discussed in the second plenary talk of CERME9. This confirms the need to address such topic in this WG. My position is that our reflection about theoretical diversity is obstructed by some beliefs that should be deconstructed and that, in order to do so, we need theoretical tools from inside and outside the mathematics education field. In this paper, my objective is to present some tools, borrowed from ATD and from Bourdieu's field theory, which I find helpful to go forward. I briefly show how I use them to consider the social dimension of theoretical multiplicity and to discuss the unifying-theories injunction, thus developing a rational discourse (*logos*) with social concerns about the issues addressed. The adjective "sociological" in the heading must be understood in this etymological meaning, this paper does not avail itself of the sociology scientific field.

Before going any further, I emphasise that, in my opinion, a valuable discussion about connecting theories relies on the participants having minimum knowledge about the theories at stake in the papers. Aside for some well-known theories, I believe it is the author's responsibility to provide the readers with a first understanding of the invoked theories. I try to do so regarding Bourdieu's field theory, assuming

that ATD is familiar enough in our research community. Hence, the second part of this text encompasses large quotes intending to provide the readers with a direct, though limited, access to the key elements of Bourdieu's analysis of science which I draw on later. Other theories appear as examples in the discussion for which I can only provide references.

THEORY, RESEARCH PRAXEOLOGY, RESEARCH PARADIGM?

In this part, I recall and connect, especially for the newcomers to theory networking like myself until recently, crucial models elaborated by the first participants in this group to address the issue of what is actually a theoretical framework. This gives me the opportunity to address the issue of what is a theory in the ATD and to discuss some points of Chevallard, Bosch and Kim's contribution to this TWG (2015). Then I propose to encompass into the theory modelling the contribution of well-identified research communities, thus considering the social dimension of networking theories.

What will we consider as networking theories in this 2015 session?

An eight-year-long joint work in CERME as well as in research projects like Remath has largely evidenced that what is at stake cannot be reduced to networking of theories considered as "organized networks of concepts (including ideas, notions, distinctions, terms, etc.) and claims about some extensive domain..." (Niss, 2007, p. 1308). Other research aspects are involved in the interconnection activities. Two directions have been proposed to model this complexity. Radford (2008) describes the concept of theory using the triplet (P, M, Q) where P is a system of basic principles, including implicit views and explicit statements, M a methodology, and Q a set of paradigmatic research questions. Hence, connecting two theories means connecting two triplets. Artigue, Bosch and Gascón (2011) use the notion of praxeology to model research theories and practices. Introduced by Chevallard as a general model for all human activities (see Bosch & Gascón, 2014, for an introduction to ATD), a punctual praxeology is a quadruplet $[T/\tau/\theta/\Theta]$ with only one type of tasks T and one associated technique τ, θ being the technology of τ, i.e. a rational discourse accounting for this technique. "The fourth component is called the "theory" and its main function is to provide a basis and support of the technological discourse" (ibid, pp. 67–68). Moreover, ATD considers more complex levels of praxeological organisations gathering punctual praxeologies which have a common technology (*local* praxeology) or a common theory (*regional* praxeology). A regional research praxeology is described through a set of research questions considered relevant when others are not, correlated techniques, their technologies and a theory. Artigue, Bosch and Gascón (2011) consider that this is the proper level to address networking issues.

What is a theory in this model? In the case of well-developed research praxeologies, the theory may fit with Niss' definition. However, not all such theories operate as identifier of their associated praxeology, because some are not recognised as "a Theory" in the research field. For instance, let us consider the so-called "double approach" (of the teachers' practices) developed by Robert and Rogalski (2002). A regional "double approach" praxeology obviously exists in mathematics education. Its theory, in both ATD and Niss' meaning, is well developed, coordinating elements from several identified theories like Theory of Conceptual Fields and Activity Theory with some more isolated concepts or results from didactics and cognitive ergonomy. Yet, there is no "Double Approach Theory", the praxeology access to social existence in the research field relies on other means, like the publication of a collective book gathering different studies (Vandebrouck, 2008) and its English translation (2013).

Now, let me emphasise that, within ATD, most praxeologies' theories are not this developed (see Chevallard et al., 2015, to go further on this issue); they may not fit with Niss' definition. One strength of this modelling of research activities is that it may be used to account for the research praxeological dynamics as Artigue and colleagues (2011, p. 2382) do: "Research praxeologies can appear as different kinds of amalgams, more or less organized depending on the maturity of the field". They highlight the part played by the technological discourse in such a stage of praxeology, when the theory of the amalgam is underdeveloped and unable to organise through a coherent whole the first results produced by the research practices. I will focus on the social dimension of the development process: the emerging praxeological organisation would not strengthen and access a certain form of social existence in the research field without the setting up of a group of researchers with common concerns, collaborating towards the development of the praxeology.

In the case of the double approach, such a group was first created around A. Robert and J. Rogalski within the Parisian laboratory Didirem, especially through the completion of several PhD theses. In 2015, the double approach community still exists; it is disseminated far beyond its original laboratory. This idea that there is no research praxeology recognised in the mathematics education field (or in some subfield) without an associated community of researchers is not accounted for by the praxeological model. Thus, I propose an extended model, called a research paradigm [1], composed of a praxeology and a correlated social organisation, working as an institution.

Connecting the three models

The praxeological model and Radford's model appear as efficient tools to account for the fact that connecting theories is not only connecting conceptual structures. They share several aspects: Q is the set of T, M the set of $[\tau/\theta]$, the explicit part of P belongs to Θ, such as does a fourth component, the set of key concepts K, added to Radford's triplet in (Bikner-Ahsbahs & Prediger, 2014). Yet each model highlights an aspect the other one overlooks. With regard to methodology, the praxeological twofold description $[\tau/\theta]$ provides an appropriate tool to consider what is happening in the case of methodological exchanges between theories (with Radford's meaning of the term), an issue addressed by (Radford, 2008, p. 322). The technique may or may not change, but certainly a new technological discourse will be produced to justify that the imported technique is consistent with the importing theory and its principles. Regarding principles, there is no place in a praxeology for the implicit part of P. This claim needs some discussion. Chevallard and colleagues (2015) argue that "a theory is made up of two main components, that we may call its "emerged part" and "immersed part. [...] In ATD, a theory is thus a hypothetical reality that assumes the form of a (necessarily fuzzy) set of explicit and implicit statements about the object of the theory." This recognises the need to encompass an implicit dimension in the human activity modelling. Yet, I dispute the idea that implicit views may be considered as parts of the praxeological *logos* component. According to the etymological meaning of this Greek term, the *lógos* is an explicit discourse. In my opinion, the praxeological model must carry this meaning where the $[\theta/\Theta]$ block refers to explicit, socially legitimised knowledge, to the *savoir* in French. However, referring to ATD and its institutional dimension, I assume that the way a praxeology lives in a given institution is determined by a set of constraints, among which culturally shared incorporated norms, many of them being implicit. Studying this implicit praxeology environment is a condition to furthering the process of developing the praxeology, as highlighted by Chevallard and colleagues (2015). As for research, the paradigm model I propose provides a tool to take into account both emerged and immersed parts: within a given paradigm, researchers' actions are regulated by the reference to the research praxeology and through the influence of the associated social organisation.

In summary, the research paradigm model presents three strong points: incorporating the different aspects of the (P, M, Q, K) and $[T/\tau/\theta/\Theta]$ models; including in the modelling project the contribution of the research community that in some cases or times plays a decisive role in the scientific identification of the research praxeology; and considering social interactions between communities within the networking issue.

LOOKING AT MATHEMATICS EDUCATION RESEARCH FROM OUTSIDE

I now present tools that I use in the last part of the paper to interpret the paradigm multiplicity in mathematics education and the injunction to unify theories.

Institutional determinations

An ATD important contribution has been to introduce the notion of ecology in mathematics education in order to fight the pedagogical voluntarisms. The mathematical and didactic praxeologies are subjected to a complex system of conditions "that cannot be reduced to those immediately identifiable in the classroom" (Bosch & Gascón, 2014, p. 72). They are constrained by a whole scale of institutional determinations among which ATD considers at the highest generic levels the influence of Civilisation and Society (ibid, p. 73). This is only one example of the crucial part given to institutions by ATD, it aims to show that this theory always immerses the addressed questions in the whole anthropological reality, with a special focus on the social organisations and the way they determine human activities. In what follows, I apply this approach to mathematics education research.

Bourdieu's field theory applied to science

A field is a structured social space, relatively autonomous from the wider social space and strongly differentiated from other fields. According to Bourdieu, science is a field. The field theory focuses on the 'closed field' dimension of these spaces, providing analysis of what is going on inside; this is the interesting contribution for our group since ATD provides adequate tools to consider external influences.

A field is characterised by a game that is played only by its agents, according to specific rules. The agents are individuals and structured groups, in science they are isolated scientists, teams or laboratories. The conformity of agents' actions to the game rules is partly controlled by objective visible means, but the key point of the theory, through the concept of *habitus*, is the inculcation of the field social rules into the agents' subjectivity. This individual system of dispositions, partly embodied as unconscious schemes, constitutes an individual's right of entry into the field.

The field game is twofold. Firstly, it is productive of something that is the field legitimised goal in the social space. The rules, and therefore the individual dispositions, are fitted to achieve the goal that every agent considers desirable. In the case of science, the goal is epistemic: accepting tacitly the existence of an objective reality endowed with some meaning and logic, scientists have the common project to understand the world and produce true statements about it. Bourdieu further adds a social dimension to the Bachelardian conception of the scientific fact construction:

> In fact, the process of knowledge validation as *legitimation* [...] concerns the relationship between the subject and the object, but also the relationship between subjects regarding the object [...]. The fact is won, constructed, observed, in and through [...] the process of verification, collective production of truth, in and through negotiation, transaction, and also homologation, ratification by the explicit expressed consensus – *homologein*. (Bourdieu, 2001/2004, pp. 72–73).

Despite this social nature, scientific homologation produces objective statements about the world thanks to specific rules of the scientific critical scrutiny, "the reference to the real, [being] constituted as the arbiter of research" (ibid, p. 69). Bourdieu also emphasises that constructed facts are all the more objective as the field is autonomous and international.

Secondly, the game is a competition between the agents, resulting in an unequal distribution of some specific form of capital, source of advantage in the game, source of power on the other agents. Thus, a field, including the scientific one, appears as:

> a structured field of forces, and also a field of struggles to conserve or to transform this field of forces. [...] It is the agents, [...] defined by the volume and structure of the specific capital they possess, that determine the structure of the field [...This one] defined by the unequal distribution of capital, bears on all the agents within it, restricting more or less the space of possible that is open to them, depending on how well placed they are within the field... (ibid, pp. 33–34)

The capital includes several species, for instance, in science, laboratory equipment, funding, and journal edition. I focus on the symbolic capital, especially on its scientific modality:

> Scientific capital is a particular kind of symbolic capital, a capital based on knowledge and recognition. (ibid, p. 34)

> A scientist's symbolic weight tends to vary with the distinctive value of his contributions and the *originality* that the competitor-peers recognize in his distinctive contribution. The notion of *visibility* [...] evokes the differential value of this capital which, concentrated in a known and recognized name, distinguishes its bearer from the undifferentiated background into which the mass of anonymous researchers merges and blurs. (ibid, pp. 55–56)

This theory of science as a field challenges an idyllic vision of the scientific community, disinterested and consensual. However, through the hypothesis of embodied dispositions, it avoids considering the scientists' participation to the capital conquest in terms of personal ambition or cynicism.

In summary, I will focus on the fact that scientific strategies are considered twofold.

> They have a pure – purely scientific- function and a social function within the field, that is to say, in relation to other agents engaged in the field. (ibid, p. 54)

> Every scientific choice is also a strategic strategy of investment oriented towards maximization of the specific, inseparably social and scientific profit offered by the field. (ibid, p. 59)

One can see a true correspondence between the triplets (institution, subjects, *assujettissements*-subjugation) of ATD (Chevallard, 1992) and (field, agents, *habitus*) of the field theory. In what follows, I consider mathematics education research as an institution immersed in and determined by a complex system of other institutions, and as a field of forces, subfield of the scientific global field.

EXTERNAL DETERMINATIONS OF THE "THEORIES ISSUE"

Research in didactics as externally determined in its questions and answers

I now consider the fact that the realm of reality of mathematics education research studies is determined by various economic, political, cultural institutions of different sizes. No one may dispute the vast distance that separates the following two objects of study: on the one hand, the passing down of arithmetic techniques in the Aymara villages of northern Chile, whose culture developed specific calculation praxeologies, and on the other hand, the use of software in the French education system to promote the learning of algebra. Is looking for universal regularities the epistemic priority of mathematics education research when, unlike physics for instance, the studied reality is so diverse? Assuming that such common phenomena exist (the didactical contract is often cited as such), which part of the two aforementioned complex realities are they able to account for? Moreover, given that the research intends to act upon the mathematics education reality, a more crucial question would be: to what extent can these regularities support engineering projects? In this paper, I will consider that adapted tools must be designed to address the problems raised by the diverse educational institutions around the world, in order to understand the dysfunctions and to produce solutions that are acceptable to these institutions and their subjects. The research questions as well as the produced answers are determined by local characteristics. The paradigm multiplicity therefore appears as a result of the epistemology of a science intending to act upon the studied reality. To take only one example, the ethnomathematics paradigm has been developed in South America as well as in Africa, as a response to a massive failure in mathematics education within educative systems that are still based on the colonial vision and present "mathematics [...] as an exclusive creation by the white race" (Gerdes, 2009, p. 31, my translation). Ethnomathematics follows as a paradigm from the need to "multiculturalise the curricula of mathematics to improve the quality of education and increase the social and cultural-self-confidence of all students". (ibid, p. 21)

Research in didactics as externally determined in its workings

Obviously, research depends on national and inter-national political and economic institutions which provide the material and the human resources. From this derives the existence of mathematics education research sub-institutions we partly find in the ICMI structure. But other institutions influence the research activities through less evident ways and means, such as cultures with more or less extended spheres of influence, up to civilisation. In spite of their scientific specific *habitus*, researchers with common culture also build upon this culture to address the research issues. That is one among other sources of some tacit principles of a paradigm. In other words, the paradigm multiplicity also results from the cultural multiplicity of the agents within the mathematics education research field. The researchers' cultural specificity may echo the educative local reality they study, hence resulting in a form of coherence and perhaps of efficiency. At the same time however, several paradigms may coexist in the same society, in the same country, investigating the same education system with different philosophical, ideological positions. As an example, let me consider ATD and the double approach that are strongly differentiated by their conception of the human being: ATD highlights the multi-institutional building of the framework within which the individual develops and acts (Chevallard, 1992, p. 91), the double approach focuses on the individual variations (Vandebrouck, 2008, p. 20). This second viewpoint is more present in the Western education research paradigms than the first one. I hypothesise that this is deeply correlated with the societies' characteristics and that it is not mere coincidence that ATD emerged in France.

Another example of external determination is the project of reducing theoretical diversity itself. This project is epistemologically founded within Bourdieu's theory since, as seen above, communication between researchers at the most international possible level is crucial in the construction of the scientific facts. However, it also comes from the requirements of political institutions, the Babel Tower aspect of research in mathematics education affecting its credibility. The proposed solution is unifying theories. Policy makers refer to the exact sciences model, and so does, rather surprisingly, mathematics education research itself, still (over)determined by its *alma mater*, mathematics. This reference neglects the diversity of educative reality. It forgets the exact sciences very long lifetime conducive to the unifying process, and that with the colonial expansion many local paradigms have simply been ignored, the occidental ones being imposed to the defeated countries. So the present homogenous theoretical landscape results as much from domination as from unification.

At this point, I have argued that the paradigm diversity is in some sense epistemologically legitimate in mathematics education and results from some social determinations of research. I have also noted that the unifying injunction might be considered as introduced into the field from outside for questionable reasons.

MATHEMATICS EDUCATION RESEARCH AS A POWER GAME

In this part, I build upon Bourdieu's statement that every scientific strategy has a social function within the field, i.e. has something to do with the distribution of power among the agents. In such a framework, the production of independent theories as well as the call for their integration in new entities are taken as contributing to the contestation and conquest of positions. For a researcher, being recognised as the creator of an identified theory clearly increases his scientific capital, much more than a less visible participation to the collective development of an existing paradigm would. This "visibility factor" fosters the paradigm multiplication, especially at the theory level; it should certainly be controlled when individual positions are at stake. However, let me now consider an emergent research community: in this case, developing a specific paradigm is an asset to free from the domination by older communities, generally tending to impose their own paradigms as ready tools which are adapted even for new problems. I will mention here the socioepistemology (Cantoral, 2013), deliberately developed by a group of Mexican researchers with the dual intent of creating tools adapted for the educative reality in South America and putting an end to what was considered as an extension of colonisation through the exclusivity of Western paradigms in didactics research.

I have already put forward that the need to unify paradigms could be epistemologically challenged by virtue of the diversity of the didactic reality depending on the societies and countries involved. Now, I question it as an obstacle to an autonomous organisation of didactical research in countries where the latter is just emerging. To finish, I reverse this point of view: if developing a paradigm is empowering for a community in the field, the call for reducing the paradigm multiplicity has something to do with relative positions of the research institutions incorporated to the paradigm. It is an aspect of the social game in the field, certainly determined by other levels of power struggles outside the scientific field as well.

CONCLUDING REMARKS

In this paper, my intent was not to contest the importance of interactions between mathematics education researchers. I recognise the crucial part of the broadest possible communication in the construction of scientific facts and the major difficulty deriving from the paradigm multiplicity in the field. My aim was to bring to light some aspects of the multidimensional complexity of this well-documented phenomenon, so far almost unexplored in this TWG. Multiplicity is an epistemological adaptation to the diversity of educational realities and a social result of symbolic power struggle within a recent research field, somehow less submitted to colonial and capitalistic rules to determine the power repartition than have been (and perhaps are) the oldest basic sciences. Hence, if reducing the number of paradigms appears as a direct solution, which favours communication thanks to a common conceptual language, this shortcut may be epistemologically inadequate for mathematics education research. Moreover, from the social point of view, it should be considered as the current hidden form of the exercise of power conquest in the field.

Unifying theories in order to produce a common discourse is not the appropriate way to scientificity for

mathematics education research in its present state: that is the opinion I have tried to convey through this text. Building on the Remath project experience among others (Artigue & Mariotti, 2014), I suggest that collaborating which brings together researchers who refer to different paradigms might be more relevant; theory networking will result from working together on the same objects. The challenge is to develop scientific research collaboration praxeologies.

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ENDNOTE

1. Using the term paradigm may be provisory. It refers to Kuhn's notion of scientific paradigm (1962). Yet, in the postscript to the second edition (1970), Kuhn writes that: "Paradigms are something shared by the members of such groups [scientific communities]" (p. 178). It seems that he does not include communities within the paradigm model.